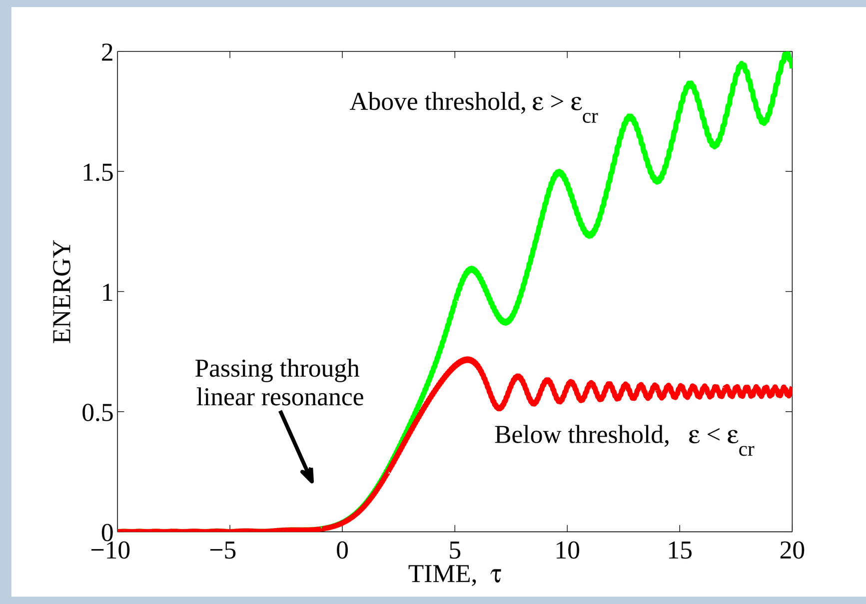
Quantum Ladder Climbing and Transition to Classical Autoresonace

Ido Barth and Lazar Friedland
The Racah institute of Physics, The Hebrew University of Jerusalem

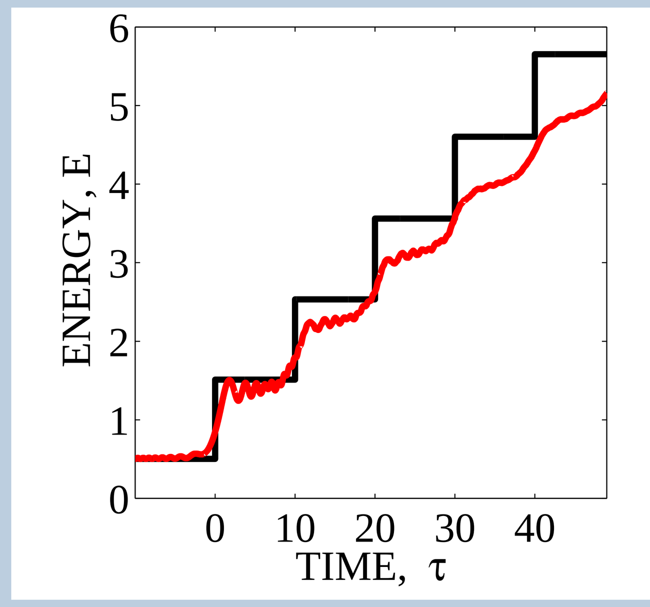
האוניברסיטה העברית בירושלים
The Hebrew University of Jerusalem

1. Introduction

Autoresonance (AR) is a generic nonlinear phase-locking phenomenon in classical dynamics. It yields a robust approach to excitation and control of nonlinear oscillatory systems by a continuous self-adjustment of the system parameters to maintain the resonance with chirped-frequency perturbations.
Ladder climbing (LC) is the quantum counterpart of the AR, characterized by continuing successive two-level Landau-Zener transitions. The AR and the LC were studied in various physical systems e.g., atoms, molecules, hydrodynamics, plasmas, Josephson circuits, etc.



Classical AR



Quantum LC

2. The chirped Anharmonic Oscillator

The simplest system that exhibit AR and LC is

$$H = \frac{1}{2}\left(p^2 + x^2\right) + \frac{1}{3}\lambda x^3 + \frac{1}{4}\beta x^4 + \varepsilon x \cos\varphi_d,$$

where $\omega_d(t) = \dot{\varphi}_d = 1 + \alpha t$. The dimensionless ($\hbar = 1$) Schrödinger equation in the energy basis of the undriven Hamiltonian reads

$$i\frac{dc_n}{dt} = E_n c_n + \varepsilon \sum_k c_k \langle \psi_k | \hat{x} | \psi_n \rangle \cos\varphi_d,$$

where the energy levels can be approximated as

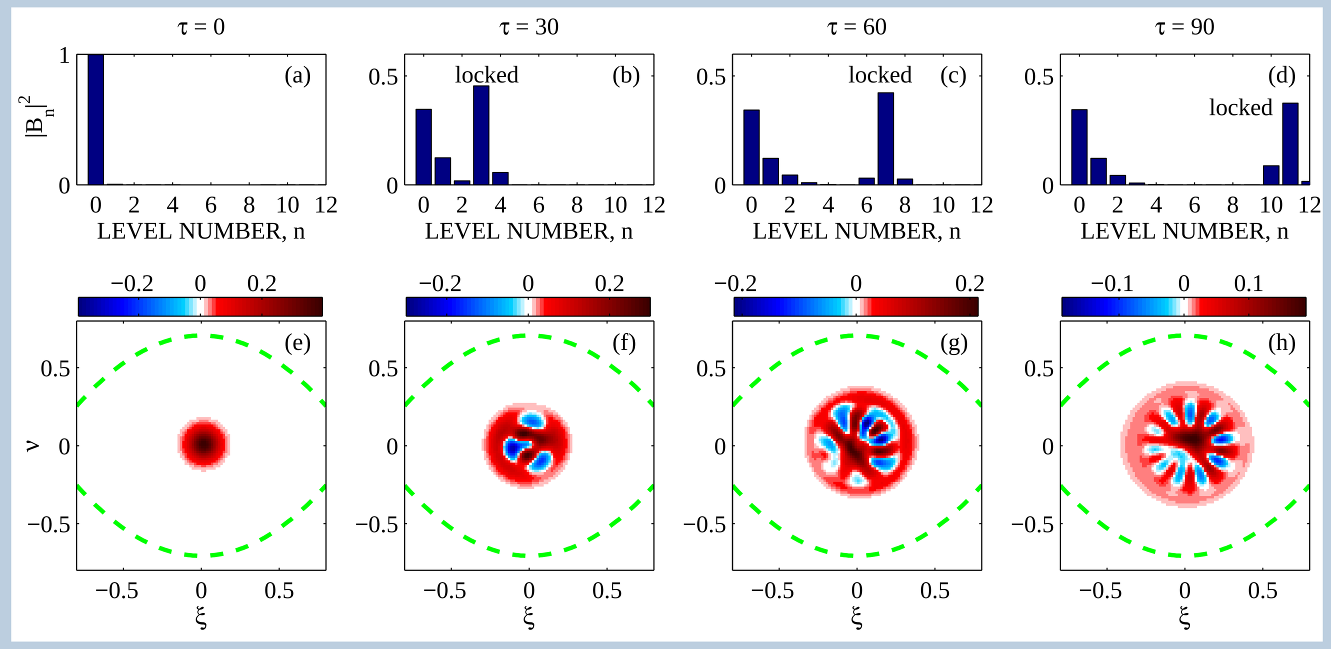
$$E_n \approx n + \frac{1}{2} + \gamma(n^2 + n) + \frac{3}{16}\beta - \frac{11}{72}\lambda^2,$$

$\gamma = \frac{3}{8}\beta - \frac{5}{12}\lambda^2$. One can define two dimensionless parameters [1]:

$$P_1 = \frac{\varepsilon}{\sqrt{2\alpha}}, \qquad P_2 = \frac{2\gamma}{\sqrt{\alpha}}$$

measuring the strength of the drive and the nonlinearity. The classicality limit is $P_2 \ll P_1 + 1$, due to overlap between successive transitions. In addition, we found that P_2 is the dimensionless Planck constant in the rotating frame [4].

3. Ladder Climbing



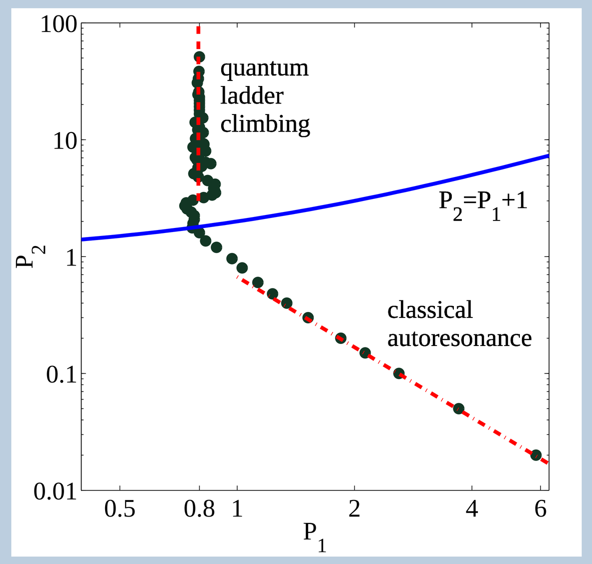
Dynamics in energy basis and in phase-space (Wigner) for $P_2 = 8$ [4].

4. Quantum to Classical Transition

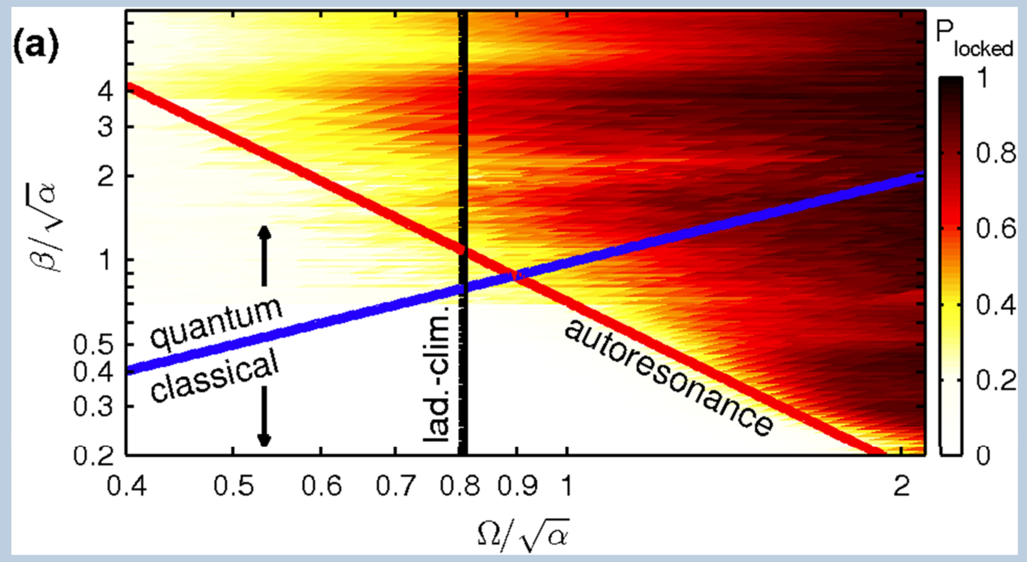
The predicted thresholds for efficient phase-locking transition are

$$P_1^{cr} = 0.8 \text{ (LC)}; \qquad P_1^{cr} = 0.82/\sqrt{P_2} \text{ (AR)}.$$

A continuous quantum-classical transition was observed in Josephson experiment by tuning the anharmonicity parameter (P_2).



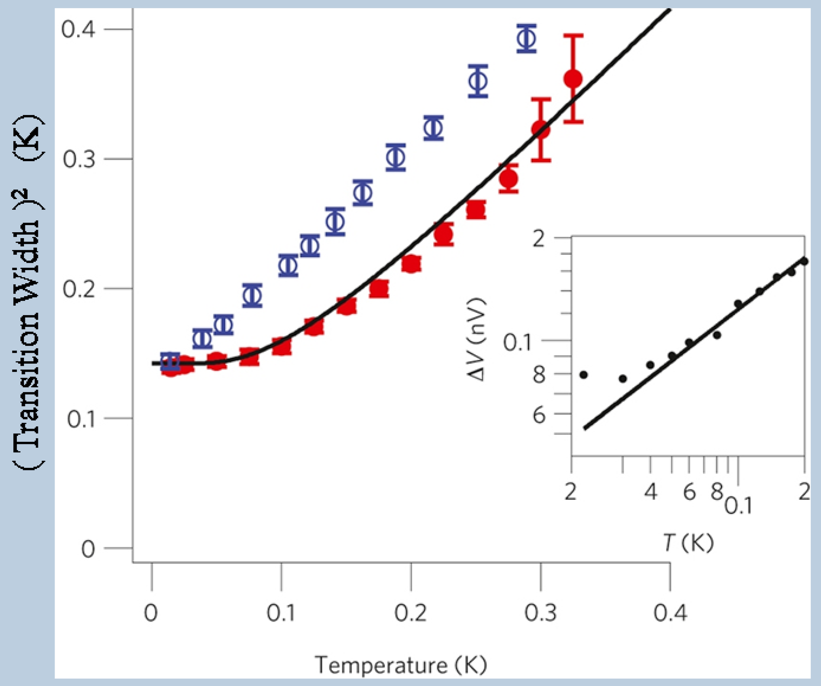
Simulations & Theory [4]



Josephson experiment [5]

5. Quantum Saturation at Low Temperatures

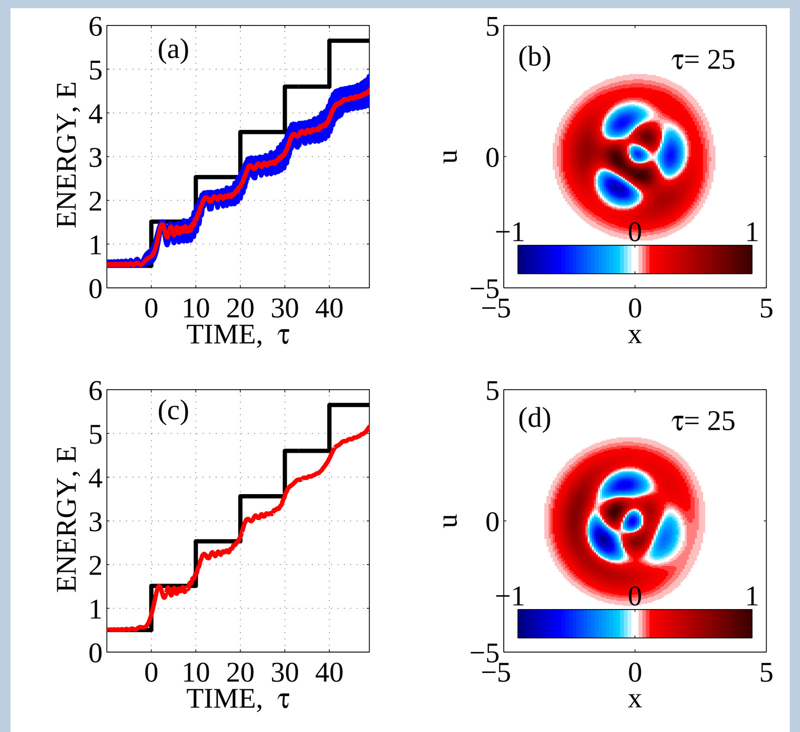
The width of the classical AR transition is $\Delta P_1 \sim \sqrt{T}$ [2]. Due to the similarity between the classical and the Wigner thermal distributions in phase-space [4], we replace T by $T_{\text{eff}} = \frac{\hbar\omega_0}{2k_B}\coth\left(\frac{\hbar\omega_0}{2k_B T}\right)$.



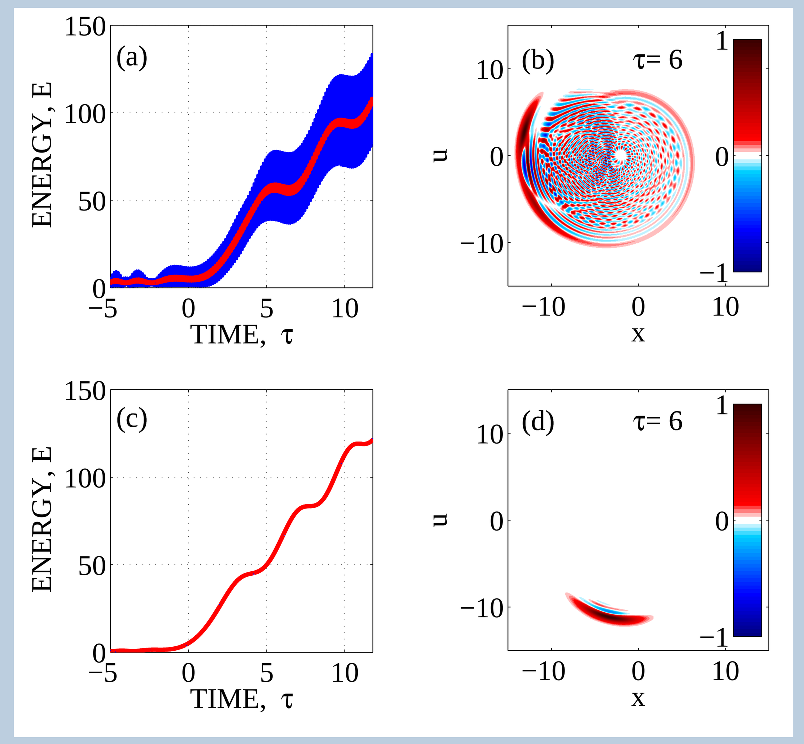
Josephson junctions experiments [3]

6. Two-photon Resonance

We found that due to an isomorphism between the chirped one- and two-photon resonances in the quantum regime, the passage through half the linear resonance, $\omega_d = \frac{1}{2} + \alpha t$, can be described similarly by replacing $\varepsilon \to \frac{8}{9}\varepsilon^2\lambda$ [6].



One- and two-photon LC [6]



One- and two-photon AR [6]

7. Conclusions

1. The chirped anharmonic oscillator is a general framework for studying, theoretically and experimentally, the quantum-classical corresponding.
2. The engineering and control of a desired state of the oscillator via the LC and AR processes can be achieved by passage through one- or two-photon resonances.
3. The quantum saturation of the threshold width, which can be tuned by adjusting α, ultimately sets the resolution of a digital detector based on autoresonance. Such a detector can be used for the readout of a quantum bit.
4. The AR threshold width can serve as a noise thermometer at low temperatures.

References & Acknowledgments

[1] G. Marcus, L. Friedland, and A. Zigler, Phys. Rev. A **69**, 013407 (2004).
[2] I. Barth, L. Friedland, E. Sarid, and A.G. Shagalov, Phys. Rev. Lett. **103**, 155001 (2009).
[3] K. W. Murch, R. Vijay, I. Barth, O. Naaman, J. Aumentado, L. Friedland, and I. Siddiqi, Nature Phys. **7**, 105 (2011).
[4] I. Barth, L. Friedland, O. Gat, and A. G. Shagalov, Phys. Rev. A 84, 013837 (2011).
[5] Y Shalibo, Y. Rofe, I. Barth, L. Friedland, R. Bialczack, J. M. Martinis, and N. Katz, Phys. Rev. Lett **108**, 037701 (2012).
[6] I. Barth and L. Friedland, Phys. Rev. A **87**, 053420 (2013).

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